

**RAY OPTICS**

12th Standard CBSE

Date : 17-Nov-22

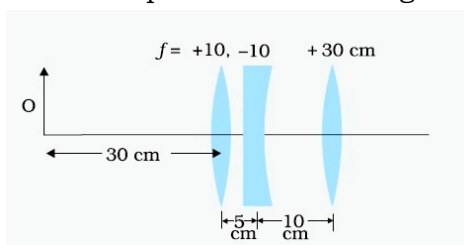
**Physics**Reg.No. : 

Exam Time : 02:00:00 Hrs

Total Marks : 100

47 x 5 = 235

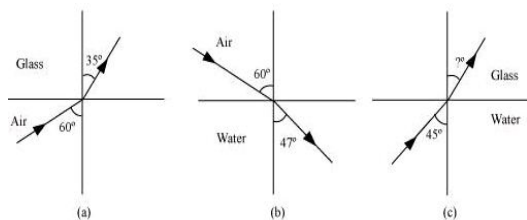
- 1) An object is placed at (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature, and magnification of the image in each case.
- 2) Suppose while sitting in a parked car, you notice a jogger approaching towards you in the side view mirror of  $R = 2$  m. If the jogger is running at a speed of  $5 \text{ m s}^{-1}$ , how fast the image of the jogger appear to move when the jogger is
- (a) 39 m,  
 (b) 29 m,  
 (c) 19 m, and  
 (d) 9 m away
- 3) (i) If  $f = 0.5$  m for a glass lens, what is the power of the lens?  
 (ii) The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass?  
 (iii) A convex lens has 20 cm focal length in air. What is focal length in water? (Refractive index of air-water = 1.33, refractive index for air-glass = 1.5.)
- 4) Find the position of the image formed by the lens combination given in the Figure.



- 5) A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?
- 6) A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.
- 7) A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water?

If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

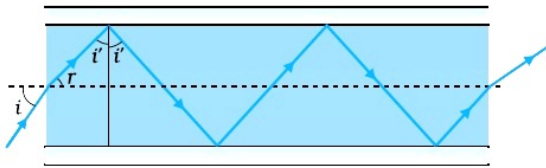
- 8) Figures (a) and (b) show refraction of a ray in air incident at  $60^\circ$  with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is  $45^\circ$  with the normal to a water-glass interface [Figure (c)]



- 9) A small bulb is placed at the bottom of a tank containing water to a depth of 80cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)
- 10) A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be  $40^\circ$ . What is the refractive index of the material of the prism? The refracting angle of the prism is  $60^\circ$ . If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.
- 11) A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is  
 (a) a convex lens of focal length 20 cm, and  
 (b) a concave lens of focal length 16 cm?
- 12) An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?
- 13) A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at  
 (a) the least distance of distinct vision (25 cm), and  
 (b) at infinity? What is the magnifying power of the microscope in each case?
- 14) A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed at 9.0mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope.
- 15) (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?  
 (b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is  $3.48 \times 10^6$  m, and the radius of lunar orbit is  $3.8 \times 10^8$  m.

- 16) Use the mirror equation to deduce that:
- an object placed between  $f$  and  $2f$  of a concave mirror produces a real image beyond  $2f$ .
  - a convex mirror always produces a virtual image independent of the location of the object.
  - the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.
  - an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.
- [Note: This exercise helps you deduce algebraically properties of images that one obtains from explicit ray diagrams.]

- 17) (a) Figure shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.
- (b) What is the answer if there is no outer covering of the pipe?



- 18) Answer the following questions:
- You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.
  - A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?
  - A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?
  - Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?
  - The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

- 19) (a) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?
- (b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.

- 20) At what angle should a ray of light be incident on the face of a prism of refracting angle  $60^\circ$  so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

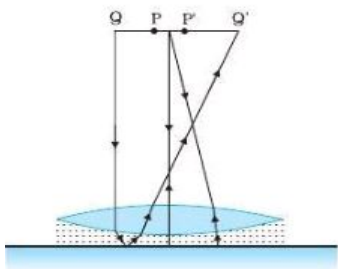
- 21) A card sheet divided into squares each of size  $1 \text{ mm}^2$  is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 9 cm) held close to the eye.
- (a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?
- (b) What is the angular magnification (magnifying power) of the lens?
- (c) Is the magnification in
- (a) equal to the magnifying power in (b) Explain?
- 
- 22) (a) At what distance should the lens be held from the figure in Exercise 9.29 in order to view the squares distinctly with the maximum possible magnifying power?
- (b) What is the magnification in this case?
- (c) Is the magnification equal to the magnifying power in this case? Explain.
- 
- 23) What should be the distance between the object in Exercise and the magnifying glass if the virtual image of each square in the figure is to have an area of  $6.25 \text{ mm}^2$ . Would you be able to see the squares distinctly with your eyes very close to the magnifier?
- [Note: Exercises will help you clearly understand the difference between magnification in absolute size and the angular magnification (or magnifying power) of an instrument.]
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- 24) Answer the following questions:
- (a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?
- (b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?
- (c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?
- (d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?
- (e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?
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- 25) An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?
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- 26) A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when
- (a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?
- (b) the final image is formed at the least distance of distinct vision (25 cm)?
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- 27) (a) For the telescope described in Exercise (a), what is the separation between the objective lens and the eyepiece?
- (b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of

the tower formed by the objective lens?

(c) What is the height of the final image of the tower if it is formed at 25 cm?

- 28) A Cassegrain telescope uses two mirrors as shown in Figure. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220mm and the small mirror is 140mm, where will the final image of an object at infinity be?

- 29) Figure shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?



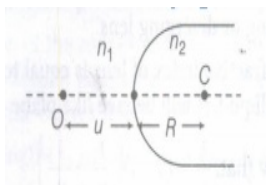
- 30) For a normal eye, the far point is at infinity and the near point of distinct vision is about 25 cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.

- 31) A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.

- (a) What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass?  
 (b) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

- 32) (i) A point object O is kept in a medium of refractive index  $n_1$  in front of a convex spherical surface of radius of curvature R which separates the second medium of refractive index  $n_2$  from the first one, as shown in the figure.

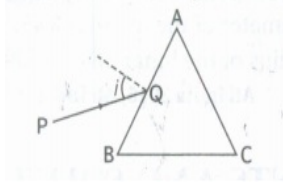
Draw the ray diagram showing the image formation and deduce the relationship between the object distance and the image distance in terms of  $n_1$ ,  $n_2$  and R.



- (ii) When the image formed above acts as a virtual object for a concave spherical surface separating the medium  $n_2$  from  $n_1$  ( $n_2 > n_1$ ) draw this ray diagram and write the similar [similar to (i)] relation. Hence obtain the expression for the lens Maker's formula.

- 33) (i) A ray PQ of light is incident on the face AB of a glass prism ABC (as shown in the figure) and emerges out of the face AC. Trace the path of the ray. Show that

$$\angle i + \angle e = \angle A + \angle \delta$$



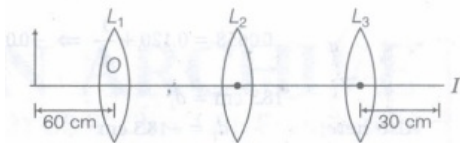
where,  $\delta$  and  $e$  denote the angle of deviation and angle of emergence, respectively. Plot a graph showing the variation of the angle of deviation as a function of angle of incidence. State the condition under which  $\angle \delta$  is minimum.

- (ii) Find out the relation between the refractive index ( $\mu$ ) of the glass prism and  $\angle A$  for the case, when the angle of prism ( $A$ ) is equal to the angle of minimum deviation ( $\delta_m$ ). Hence, obtain the value of the refractive index for angle of prism  $A = 60^\circ$ .

- 34) Define magnifying power of a telescope. Write its expression. A small telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If this telescope is used to view a 100 m high tower 3 km away, find the height of the final image, when it is formed 25 cm away from the eyepiece.

- 35) (i) Draw a labeled ray diagram showing the formation of a final image by a compound microscope at least distance of distinct vision.
- (ii) The total magnification produced by a compound microscope is 20. The magnification produced by the eyepiece is 5. The microscope is focused on a certain object. The distance between the object and eyepiece is observed to be 14 cm. If least distance of distinct vision is 20 cm, calculate the focal length of the object and the eyepiece.

- 36) (i) Explain with reason, how the power of a diverging lens changes when
- (a) it is kept in a medium of refractive index greater than that of the lens.
- (b) incident red light is replaced by violet light.
- (ii) Three lenses  $L_1$ ,  $L_2$  and  $L_3$  each of focal length 30 cm are placed coaxially as shown in the figure. An object is held at 60 cm from the optic centre of lens  $L_1$ . The final real image is formed at the focus of  $L_3$ . Calculate the separation between (a) ( $L_1$  and  $L_2$ ) and (b) ( $L_2$  and  $L_3$ ).



- 37) (i) Deduce the expression by drawing a suitable ray diagram for the refractive index of a triangular glass prism in terms of the angle of minimum deviation ( $D$ ) and the angle of prism ( $A$ ) Draw a plot showing the variation of the angle of deviation with the angle of incidence.
- (ii) Calculate the value of the angle of incidence when a ray of light incident on one face of an equilateral glass prism produces the emergent ray, which just grazes along the adjacent face. Refractive index of the prism is  $\sqrt{2}$ .

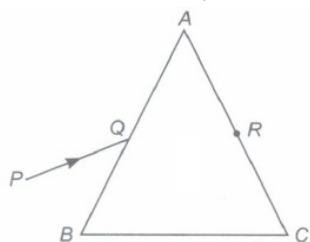
38) A thin convex lens having two surfaces of radii of curvature  $R_1$  and  $R_2$  is made of a material of refractive index  $\mu_2$ . It is kept in a medium of refractive index  $\mu_1$ . Derive, with the help of a ray diagram, the lens maker's formula when a point object placed on the principal axis in front of the radius of curvature  $R_1$  produces an image I on the other side of the lens.

39) How would you estimate rough focal length of a converging lens?

Draw a ray diagram to show image formation by a diverging lens. Using this diagram, derive the relation between object distance  $u$ , image distance  $v$  and focal length of the lens. Sketch the graph between  $1/u$  and  $1/v$  for this lens.

40) (a) Two thin convex lenses  $L_1$  and  $L_2$  of focal lengths  $f_1$  and  $f_2$  respectively, are placed co-axially in contact. An object is placed at a point beyond the focus of lens  $L_1$ . Draw a ray diagram to show the image formation by the combination and hence derive the expression for the focal length of the combined system.

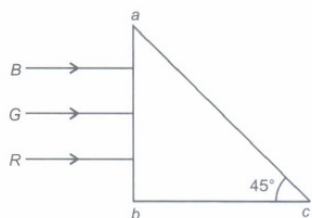
(b) A ray PQ incident on the face AB of a prism ABC, as shown in the figure, emerges from the face AC such that  $AQ = AR$ .



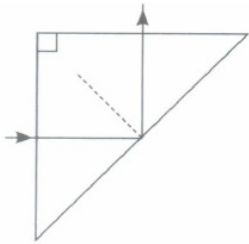
Draw the ray diagram showing the passage of the ray through the prism. If the angle of the prism is  $60^\circ$  and refractive index of the material of the prism is  $\sqrt{3}$ , determine the values of angle of incidence and angle of deviation.

41) (a) A ray of monochromatic light is incident on one of the faces of an equilateral triangular prism of refracting angle  $A$ . Trace the path of ray passing through the prism. Hence derive an expression for the refractive index of the material of the prism in terms of the angle of minimum deviation and its refracting angle.

(b) Three light rays red (R), green (G) and blue (B) are incident on the right angled prism abc at face ab. The refractive indices of the material of the prism for red, green and blue wavelengths are respectively 1.39, 1.44 and 1.47. Trace the paths of these rays reasoning out the difference in their behaviour.



42) (i) Plot a graph to show variation of the angle of deviation as a function of angle of incidence for light passing through a prism. Derive an expression for refractive index of the prism in terms of angle of minimum deviation and angle of prism.



(ii) What is dispersion of light? What is its cause?

(iii) A ray of light incident normally on one face of a right isosceles prism is totally reflected as shown in figure. What must be the minimum value of refractive index of glass? Give relevant calculations.

- 43) (a) Draw a ray diagram showing image formation in a compound microscope. Define the term 'limit of resolution' and name the factors on which it depends. How is it related to resolving power of microscope?  
 (b) Suggest two ways by which the resolving power of a microscope can be increased.  
 (c) "A telescope resolves whereas a microscope magnifies." Justify this statement.

- 44) (a) Draw a ray diagram for the formation of image by a compound microscope. Define its magnifying power. Deduce the expression for the magnifying power of the microscope.  
 (b) Explain:  
 (i) why must both the objective and the eyepiece of a compound microscope have short focal lengths?  
 (ii) while viewing through a compound microscope, why should our eyes be positioned not on the eyepiece but a short distance away from it for best viewing?

- 45) Draw a ray diagram showing the image formation of a distant object by a refracting telescope. Define its magnifying power and write the two important factors considered to increase the magnifying power. Describe briefly the two main limitations and explain how far these can be minimized in a reflecting telescope.

- 46) Draw a labelled ray diagram of an astronomical telescope for the near point adjustment. You are given three lenses of powers 0.5 D, 4 D, 10 D. State, with reason, which two lenses will you select for constructing a good astronomical telescope. Calculate the resolving power of this telescope, assuming the diameter of the objective lens to be 6 cm and the wavelength of light used to be 540 nm.

- 47) Draw an astronomical telescope, when the final image is formed at the least distance of distinct vision (D) from the eye. Define the magnifying power of the astronomical telescope and derive its formula.

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1) The focal length  $f = -15/2 \text{ cm} = -7.5 \text{ cm}$

(i) The object distance  $u = -10 \text{ cm}$ . Then gives

$$\frac{1}{v} + \frac{1}{10} = \frac{1}{-7.5}$$

$$\text{or } v = \frac{10 \times 7.5}{-2.5} = -30 \text{ cm}$$

The image is 30 cm from the mirror on the same side as the object

$$\text{Also, magnification } m = \frac{v}{u} = -\frac{(-30)}{(-10)} = -3$$

The image is magnified, real and inverted.

(ii) The object distance  $u = -5 \text{ cm}$ . Then from

$$\frac{1}{v} + \frac{1}{-5} = \frac{1}{-7.5}$$

$$\text{or } v = \frac{5 \times 7.5}{(7.5-5)} = 15 \text{ cm}$$

This image is formed at 15 cm behind the mirror. It is a virtual image.

$$\text{Magnification } m = 15 - \frac{v}{u} = -\frac{15}{(-5)} = 3$$

The image is magnified, virtual and erect.

2)

From the mirror equation, we get

$$v = \frac{fu}{u-f}$$

For convex mirror, since  $R = 2 \text{ m}$ ,  $f = 1 \text{ m}$ . Then

$$\text{for } u = -39 \text{ m}, v = \frac{(-39) \times 1}{-39-1} = \frac{39}{40} \text{ m}$$

Since the jogger moves at a constant speed of  $5 \text{ m s}^{-1}$ , after 1 s the position of the image  $v$  (for  $u = -39 + 5 = -34$ ) is  $(34/35) \text{ m}$ . The shift in the position of image in 1 s is

$$\frac{39}{40} - \frac{34}{35} = \frac{1365-1360}{1400} = \frac{5}{1400} = \frac{1}{280}$$

Therefore, the average speed of the image when the jogger is between 39 m and 34 m from the mirror, is  $(1/280) \text{ m s}^{-1}$

Similarly, it can be seen that for  $u = -29 \text{ m}$ ,  $-19 \text{ m}$  and  $-9 \text{ m}$ , the

speed with which the image appears to move is

$$\frac{1}{150} \text{ ms}^{-1}, \frac{1}{60} \text{ ms}^{-1} \text{ and } \frac{1}{10} \text{ ms}^{-1}$$

Although the jogger has been moving with a constant speed, the speed of his/her image appears to increase substantially as he/she moves closer to the mirror. This phenomenon can be noticed by any person sitting in a stationary car or a bus.

In case of moving vehicles, a similar phenomenon could be observed if the vehicle in the rear is moving closer with a constant speed.

3) (i) Power = +2 dioptre.

(ii) Here, we have  $f = +12$  cm,  $R_1 = +10$  cm,  $R_2 = -15$  cm.

Refractive index of air is taken as unity.

We use the lens formula. The sign convention has to be applied for  $f$ ,  $R_1$  and  $R_2$ .

Substituting the values, we have

$$\frac{1}{12} = (n - 1) \left( \frac{1}{10} - \frac{1}{15} \right)$$

This gives  $n = 1.5$ .

(iii) For a glass lens in air,  $n_2 = 1.5$ ,  $n_1 = 1$ ,  $f = +20$  cm. Hence, the lens formula gives

$$\frac{1}{20} = 0.5 \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

For the same glass lens in water,  $n_2 = 1.5$ ,  $n_1 = 1.33$ . Therefore

$$\frac{1.33}{f} = (1.5 - 1.33) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

Combining these two equations, we find  $f = +78.2$  cm.

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4)

Image formed by the first lens

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10}$$

$$\text{or } v_1 = 15 \text{ cm}$$

The image formed by the first lens serves as the object for the second.

This is at a distance of  $(15 - 5)$  cm = 10 cm to the right of the second lens. Though the image is real, it serves as a virtual object for the second lens, which means that the rays appear to come from it for the second lens.

$$\frac{1}{v_2} - \frac{1}{10} = \frac{1}{-10}$$

$$\text{or } v_2 = \infty$$

The virtual image is formed at an infinite distance to the left of the second lens. This acts as an object for the third lens.

$$\frac{1}{v_3} - \frac{1}{u_3} = \frac{1}{f_3}$$

$$\text{or } \frac{1}{v_3} = \frac{1}{\infty} + \frac{1}{30}$$

$$\text{or } v_3 = 30 \text{ cm}$$

The final image is formed 30 cm to the right of the third lens.

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5)

Size of the candle,  $h = 2.5$  cm

Image size =  $h'$

Object distance,  $u = -27$  cm

Radius of curvature of the concave mirror,  $R = -36$  cm

$$f = \frac{R}{2} = -18 \text{ cm}$$

Image distance =  $v$

The image distance can be obtained using the mirror formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\frac{1}{-18} = \frac{1}{-27} = \frac{-3+2}{54} = -\frac{1}{54}$$

$$\therefore v = -54 \text{ cm}$$

Therefore, the screen should be placed 54 cm away from the mirror to obtain a sharp image.

The magnification of the image is given as:

$$m = \frac{h'}{h} = -\frac{v}{u}$$

$$\therefore h' = -\frac{v}{u} \times h$$

$$= -\left(\frac{-54}{-27}\right) \times 2.5 = -5 \text{ cm}$$

The height of the candle's image is 5 cm. The negative sign indicates that the image is inverted and real.

If the candle is moved closer to the mirror, then the screen will have to be moved away from the mirror in order to obtain the image.

6)

Height of the needle,  $h_1 = 4.5$  cm

Object distance,  $u = -12$  cm

Focal length of the convex mirror,  $f = 15$  cm

Image distance =  $v$

The value of  $v$  can be obtained using the mirror formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60}$$

$$\therefore v = \frac{60}{9} = 6.7 \text{ cm}$$

Hence, the image of the needle is 6.7 cm away from the mirror. Also, it is on the other side of the mirror.

The image size is given by the magnification formula:

$$m = \frac{h_2}{h_1} = -\frac{v}{u}$$

$$\therefore h_2 = -\frac{v}{u} \times h_1$$

$$= \frac{-6.7}{-12} \times 4.5 = +2.5 \text{ cm}$$

$$\text{Hence, magnification of the image, } m = \frac{h_2}{h_1} = \frac{2.5}{4.5} = 0.56$$

The height of the image is 2.5 cm. The positive sign indicates that the image is erect, virtual, and diminished.

If the needle is moved farther from the mirror, the image will also move away from the mirror, and the size of the image will reduce gradually.

7)

Actual depth of the needle in water,  $h_1 = 12.5$  cm

Apparent depth of the needle in water,  $h_2 = 9.4$  cm

Refractive index of water =  $\mu$

The value of  $\mu$  can be obtained as follows:

$$\begin{aligned}\mu &= \frac{h_2}{h_1} \\ &= \frac{12.5}{9.4} \approx 1.33\end{aligned}$$

Hence, the refractive index of water is about 1.33.

Water is replaced by a liquid of refractive index,  $\mu' = 1.63$

The actual depth of the needle remains the same, but its apparent depth changes. Let  $y$  be the new apparent depth of the needle. Hence, we can write the relation:

$$\begin{aligned}\mu' &= \frac{h_1}{y} \\ \therefore y &= \frac{h_1}{\mu'} \\ &= \frac{12.5}{1.63} = 7.67 \text{ cm}\end{aligned}$$

Hence, the new apparent depth of the needle is 7.67 cm. It is less than  $h_2$ . Therefore, to focus the needle again, the microscope should be moved up.

$$\begin{aligned}\therefore \text{Distance by which the microscope should be moved up} &= 9.4 - 7.67 \\ &= 1.73 \text{ cm}\end{aligned}$$


---

8) As per the given figure, for the glass-air interface:

Angle of incidence,  $i = 60^\circ$

Angle of refraction,  $r = 35^\circ$

The relative refractive index of glass with respect to air is given by Snell's law as:

$$\begin{aligned}\mu_g^a &= \frac{\sin i}{\sin r} \\ &= \frac{\sin 60^\circ}{\sin 35^\circ} = \frac{0.8660}{0.5736} = 1.51 \dots(1)\end{aligned}$$

As per the given figure, for the air-water interface:

Angle of incidence,  $i = 60^\circ$

Angle of refraction,  $r = 47^\circ$

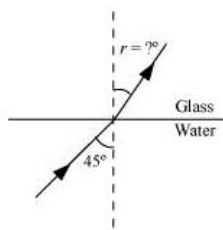
The relative refractive index of water with respect to air is given by Snell's law as:

$$\begin{aligned}\mu_w^a &= \frac{\sin i}{\sin r} \\ &= \frac{\sin 60}{\sin 47} = \frac{0.8660}{0.7314} = 1.184 \dots(2)\end{aligned}$$

Using (1) and (2), the relative refractive index of glass with respect to water can be obtained as:

$$\mu_g^w = \frac{\mu_g^a}{\mu_w^a} = \frac{1.51}{1.84} = 1.275$$

The following figure shows the situation involving the glass-water interface.



Angle of incidence,  $i = 45^\circ$

Angle of refraction =  $r$

From Snell's law,  $r$  can be calculated as:

$$\begin{aligned}\frac{\sin i}{\sin r} &= \mu_g^w \\ &= \frac{\sin 45^\circ}{\sin r} = 1.275\end{aligned}$$

$$\sin r = \frac{\frac{1}{\sqrt{2}}}{1.275} = 0.5546$$

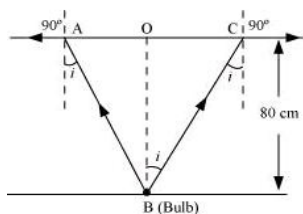
$$\therefore r = \sin^{-1}(0.5546) = 38.68^\circ$$

Hence, the angle of refraction at the water-glass interface is  $38.68^\circ$ .

9) Actual depth of the bulb in water,  $d_1 = 80 \text{ cm} = 0.8 \text{ m}$

Refractive index of water,  $\mu = 1.33$

The given situation is shown in the following figure:



Where,

$i$  = Angle of incidence

$r$  = Angle of refraction =  $90^\circ$

Since the bulb is a point source, the emergent light can be considered as a circle of radius,  $R = \frac{AC}{2} = OA = OB$

Using Snell's law, we can write the relation for the refractive index of water as:

$$\mu = \frac{\sin i}{\sin r}$$

$$1.33 = \frac{\sin 90^\circ}{\sin i}$$

$$\therefore i = \sin^{-1} \left( \frac{1}{1.33} \right) = 48.75^\circ$$

Using the given figure, we have the relation:

$$\tan i = \frac{OC}{OB} = \frac{R}{d_1}$$

$$\therefore R = \tan 48.75^\circ \times 0.8 = 0.91 \text{ m}$$

$$\therefore \text{Area of the surface of water} = \pi R^2 = \pi (0.91)^2 = 2.61 \text{ m}^2$$

Hence, the area of the surface of water through which the light from the bulb can emerge is approximately  $2.61 \text{ m}^2$ .

10) Angle of minimum deviation,  $\delta'_m = 40^\circ$

Angle of the prism,  $A = 60^\circ$

Refractive index of water,  $\mu = 1.33$

Refractive index of the material of the prism =  $\mu'$

The angle of deviation is related to refractive index ( $\mu'$ ) as:

$$\begin{aligned}\mu' &= \frac{\sin \frac{(A + \delta'_m)}{2}}{\sin \frac{A}{2}} \\ &= \frac{\sin \frac{(60^\circ + 40^\circ)}{2}}{\sin \frac{60^\circ}{2}} = \frac{\sin 50^\circ}{\sin 30^\circ} = 1.532\end{aligned}$$

Hence, the refractive index of the material of the prism is 1.532.

Since the prism is placed in water, let be the new angle of minimum deviation for the same prism.

The refractive index of glass with respect to water is given by the relation:

$$\begin{aligned}\mu_g^w &= \frac{\mu'}{\mu} = \frac{\sin \frac{(A + \delta'_m)}{2}}{\sin \frac{A}{2}} \\ &= \frac{\sin \frac{(A + \delta'_m)}{2}}{\sin \frac{A}{2}} = \frac{\mu'}{\mu} \sin \frac{A}{2} \\ &= \frac{\sin \frac{(A + \delta'_m)}{2}}{\sin \frac{A}{2}} = \frac{1.532}{1.33} \times \sin \frac{60^\circ}{2} = 0.5759 \\ &= \frac{\sin \frac{(A + \delta'_m)}{2}}{\sin \frac{A}{2}} = \sin^{-1} 0.5759 = 35.16^\circ \\ &= 60^\circ + \delta'_m = 70.32^\circ \\ \therefore \delta'_m &= 70.32^\circ - 60^\circ = 10.32^\circ\end{aligned}$$

Hence, the new minimum angle of deviation is  $10.32^\circ$ .

11) In the given situation, the object is virtual and the image formed is real.

Object distance,  $u = +12$  cm

(a) Focal length of the convex lens,  $f = 20$  cm

Image distance =  $v$

According to the lens formula, we have the relation:

$$\begin{aligned}\frac{1}{v} - \frac{1}{u} &= \frac{1}{f} \\ \frac{1}{v} - \frac{1}{12} &= \frac{1}{20} \\ \frac{1}{v} &= \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60} = \frac{8}{60} \\ \therefore v &= \frac{60}{8} = 7.5\end{aligned}$$

Hence, the image is formed 7.5 cm away from the lens, toward its right.

(b) Focal length of the concave lens,  $f = -16$  cm

Image distance =  $v$

According to the lens formula, we have the relation:

$$\begin{aligned}\frac{1}{v} - \frac{1}{u} &= \frac{1}{f} \\ \frac{1}{v} &= -\frac{1}{16} + \frac{1}{12} = \frac{-3+4}{48} = \frac{1}{48} \\ \therefore v &= 48\text{cm}\end{aligned}$$

Hence, the image is formed 48 cm away from the lens, toward its right.

12)

Size of the object,  $h_1 = 3$  cmObject distance,  $u = -14$  cmFocal length of the concave lens,  $f = -21$  cmImage distance =  $v$ 

According to the lens formula, we have the relation:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = -\frac{1}{21} - \frac{1}{14} = \frac{-2-3}{42} = \frac{-5}{42}$$

$$\therefore v = -\frac{42}{5} = -8.4 \text{ cm}$$

Hence, the image is formed on the other side of the lens, 8.4 cm away from it. The negative sign shows that the image is erect and virtual.

The magnification of the image is given as:

$$m = \frac{\text{Image height } (h_2)}{\text{Object height } (h_1)} = \frac{-8.4}{-14}$$

$$\therefore h_2 = \frac{-8.4}{-14} \times 3 = 0.6 \times 3 = 1.8 \text{ cm}$$

Hence, the height of the image is 1.8 cm

If the object is moved further away from the lens, then the virtual image will move toward the focus of the lens, but not beyond it. The size of the image will decrease with the increase in the object distance.

13)

Focal length of the objective lens,  $f_1 = 2.0$  cmFocal length of the eyepiece,  $f_2 = 6.25$  cmDistance between the objective lens and the eyepiece,  $d = 15$  cm(a) Least distance of distinct vision,  $d' = 25$  cm $\therefore$  Image distance for the eyepiece,  $v_2 = -25$  cmObject distance for the eyepiece =  $u_2$ 

According to the lens formula, we have the relation

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{u_2} = \frac{1}{v_2} - \frac{1}{f_2}$$

$$= \frac{1}{-25} - \frac{1}{6.25} = \frac{-1-4}{25} = \frac{-5}{25}$$

$$\therefore u_2 = -5 \text{ cm}$$

Image distance for the objective lens,

$$v_1 = d + u_2 = 15 - 5 = 10 \text{ cm}$$

Object distance for the objective lens =  $u_1$ 

According to the lens formula, we have the relation:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1}$$

$$= \frac{1}{10} - \frac{1}{2} = \frac{1-5}{10} = \frac{-4}{10}$$

$$\therefore u_1 = -2.5 \text{ cm}$$

Magnitude of the object distance,  $|u_1| = 2.5$  cm

The magnifying power of a compound microscope is given by the relation:

$$m = \frac{v_1}{|u_1|} \left( 1 + \frac{d'}{f_2} \right)$$



$$= \frac{10}{2.5} \left( 1 + \frac{25}{6.25} \right) = 4(1 + 4) = 20$$

Hence, the magnifying power of the microscope is 20.

(b) The final image is formed at infinity.

∴ Image distance for the eyepiece,  $v_2 = \infty$

Object distance for the eyepiece =  $u_2$

According to the lens formula, we have the relation:

$$\frac{1}{u_2} = \frac{1}{v_2} - \frac{1}{f_2}$$

$$\frac{1}{\infty} - \frac{1}{u_2} = \frac{1}{6.25}$$

∴  $u_2 = -6.25$  cm

Image distance for the objective lens,

$$v_1 = d + u_2 = 15 - 6.25 = 8.75 \text{ cm}$$

Object distance for the objective lens =  $u_1$

According to the lens formula, we have the relation:

$$\frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1}$$

$$\frac{1}{u_1} = \frac{1}{8.75} - \frac{1}{2.0}$$

$$= \frac{1}{8.75} - \frac{1}{2.0} = \frac{2 - 8.75}{17.5}$$

Magnitude of the object distance,  $|u_1| = 2.59$  cm

The magnifying power of a compound microscope is given by the relation:

$$m = \frac{v_1}{|u_1|} \left( \frac{d'}{|u_2|} \right)$$

$$= \frac{8.75}{2.59} \times \frac{25}{6.25} = 13.51$$

Hence, the magnifying power of the microscope is 13.51.

- 14) Focal length of the objective lens,  $f_o = 8 \text{ mm} = 0.8 \text{ cm}$   
 Focal length of the eyepiece,  $f_e = 2.5 \text{ cm}$   
 Object distance for the objective lens,  $u_o = -9.0 \text{ mm} = -0.9 \text{ cm}$   
 Least distance of distant vision,  $d = 25 \text{ cm}$   
 Image distance for the eyepiece,  $v_e = -d = -25 \text{ cm}$   
 Object distance for the eyepiece =  $u_c$

Using the lens formula, we can obtain the value of  $u_2$  as:

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= \frac{1}{-25} - \frac{1}{2.5} = \frac{-1-10}{25} = \frac{-11}{25}$$

$$\therefore u_c = -\frac{25}{11} = -2.27 \text{ cm}$$

We can also obtain the value of the image distance for the objective lens ( $v_o$ ) using the lens formula.

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o}$$

$$= \frac{1}{0.8} - \frac{1}{0.9} = \frac{0.9-0.8}{0.72} = \frac{0.1}{0.72}$$

$$\therefore v_o = 7.2 \text{ cm}$$

The distance between the objective lens and the eyepiece

$$= |u_c| + v_o$$

$$= 2.27 + 7.2$$

$$= 9.47 \text{ cm}$$

The magnifying power of the microscope is calculated as:

$$\frac{v_o}{|u_o|} \left( 1 + \frac{d'}{f_e} \right)$$

$$= \frac{7.2}{0.9} \left( 1 + \frac{25}{2.5} \right) = 8(1 + 10) = 88$$

Hence, the magnifying power of the microscope is 88.

- 15) Focal length of the objective lens,  $f_o = 15 \text{ m} = 15 \times 10^2 \text{ cm}$

Focal length of the eyepiece,  $f_e = 1.0 \text{ cm}$

(a) The angular magnification of a telescope is given as:

$$\alpha = \frac{f_o}{f_e}$$

$$= \frac{15 \times 10^2}{1.0} = 1500$$

Hence, the angular magnification of the given refracting telescope is 1500.

(b) Diameter of the moon,  $d = 3.48 \times 10^6 \text{ m}$

Let  $d'$  be the diameter of the image of the moon formed by the objective lens.

Radius of the lunar orbit,  $r_0 = 3.8 \times 10^8 \text{ m}$

The angle subtended by the diameter of the moon is equal to the angle subtended by the image.

$$\frac{d}{r_0} = \frac{d'}{f_o}$$

$$\frac{3.48 \times 10^6}{3.8 \times 10^8} = \frac{d'}{15}$$

$$\therefore d' = \frac{3.48}{3.8} \times 10^{-2} \times 15$$

$$= 13.74 \times 10^{-2} \text{ m} = 13.74 \text{ cm}$$

Hence, the diameter of the moon's image formed by the objective lens is 13.74 cm.

- 16) (a) For a concave mirror, the focal length ( $f$ ) is negative.

$$\therefore f < 0$$

When the object is placed on the left side of the mirror, the object distance ( $u$ ) is negative.

$$\therefore u < 0$$

For image distance  $v$ , we can write the lens formula as:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \dots(1)$$

The object lies between  $f$  and  $2f$ .

$$\therefore 2f < u < f (\because u \text{ and } f \text{ are negative})$$

$$\frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$$

$$-\frac{1}{2f} < -\frac{1}{u} < \frac{1}{f}$$

$$\frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < 0 \dots(2)$$

Using equation (1), we get:

$$\frac{1}{2f} < \frac{1}{v}$$

$$2f > v$$

$$-v > -2f$$

Therefore, the image lies beyond  $2f$ .

(b) For a convex mirror, the focal length ( $f$ ) is positive.

$$\therefore f > 0$$

When the object is placed on the left side of the mirror, the object distance ( $u$ ) is negative.

$$\therefore u < 0$$

For image distance  $v$ , we have the mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

Using equation (2), we can conclude that:

$$\frac{1}{v} < 0$$

$$v > 0$$

Thus, the image is formed on the back side of the mirror.

Hence, a convex mirror always produces a virtual image, regardless of the object distance.

(c) For a convex mirror, the focal length (f) is positive.

$$\therefore f > 0$$

When the object is placed on the left side of the mirror, the object distance (u) is negative,

$$\therefore u < 0$$

For image distance v, we have the mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

But we have  $u < 0$

$$\therefore \frac{1}{v} > \frac{1}{f}$$

$$v < f$$

Hence, the image formed is diminished and is located between the focus (f) and the pole.

(d) For a concave mirror, the focal length (f) is negative.

$$\therefore f < 0$$

When the object is placed on the left side of the mirror, the object distance (u) is negative.

$$\therefore u < 0$$

It is placed between the focus (f) and the pole.

$$\therefore f > u > 0$$

$$\frac{1}{f} < \frac{1}{u} < 0$$

$$\frac{1}{f} - \frac{1}{u} < 0$$

For image distance v, we have the mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\therefore \frac{1}{v} < 0$$

$$v > 0$$

The image is formed on the right side of the mirror. Hence, it is a virtual image

For  $u < 0$  and  $v > 0$ , we can write:

$$\frac{1}{u} > \frac{1}{v}$$

$$v > u$$

$$\text{Magnification, } m = \frac{v}{u} > 1$$

Hence, the formed image is enlarged.

17)

(a) Refractive index of the glass fibre,  $\mu_1 = 1.68$ Refractive index of the outer covering of the pipe,  $\mu_2 = 1.44$ Angle of incidence =  $i$ Angle of refraction =  $r$ Angle of incidence at the interface =  $i'$ The refractive index ( $\mu$ ) of the inner core – outer core interface is given a

$$\mu = \frac{\mu_1}{\mu_2} = \frac{1}{\sin i}$$

$$\sin i' = \frac{\mu_1}{\mu_2}$$

$$= \frac{1.44}{1.68} = 0.8571$$

$$\therefore i' = 59^\circ$$

For the critical angle, total internal reflection (TIR) takes place only when  $i > i'$ , i.e.,  $i > 59^\circ$ Maximum angle of reflection,  $r_{\max} = 90^\circ - i' = 90^\circ - 59^\circ = 31^\circ$ 

Let,

 $i_{\max}$  be the maximum angle of incidence.The refractive index at the air - glass interface,  $\mu_1 = 1.68$ 

We have the relation for the maximum angles of incidence and reflection as:

$$\mu_1 = \frac{\sin i_{\max}}{\sin r_{\max}}$$

$$\sin i_{\max} = \mu_1 \sin r_{\max}$$

$$= 1.68 \sin 31^\circ$$

$$= 1.68 \times 0.5150$$

$$= 0.8652$$

$$\therefore i_{\max} = \sin^{-1} 0.8652 \approx 60^\circ$$

Thus, all the rays incident at angles lying in the range  $0 < i < 60^\circ$  will suffer total internal reflection.

(b) If the outer covering of the pipe is not present, then:

Refractive index of the outer pipe,  $\mu_1 =$  Refractive index of air = 1For the angle of incidence  $i = 90^\circ$ , we can write Snell's law at the air – pipe interface as:

$$\frac{\sin i}{\sin r} = \mu_2 = 1.68$$

$$\sin r = \frac{\sin 90^\circ}{1.68} = \frac{1}{1.68}$$

$$r = \sin^{-1} (0.5952)$$

$$= 36.5^\circ$$

$$\therefore i' = 90^\circ - 36.5^\circ = 53.5^\circ$$

Since  $i' > r$ , all incident rays will suffer total internal reflection.

18)

(a) Yes

Plane and convex mirrors can produce real images as well. If the object is virtual, i.e., if the light rays converging at a point behind a plane mirror (or a convex mirror) are reflected to a point on a screen placed in front of the mirror, then a real image will be formed.

(b) No

A virtual image is formed when light rays diverge. The convex lens of the eye causes these divergent rays to converge at the retina. In this case, the virtual image serves as an object for the lens to produce a real image.

(c) The diver is in the water and the fisherman is on land (i.e., in air). Water is a denser medium than air. It is given that the diver is viewing the fisherman. This indicates that the light rays are travelling from a denser medium to a rarer medium. Hence, the refracted rays will move away from the normal. As a result, the fisherman will appear to be taller.

(d) Yes; Decrease

The apparent depth of a tank of water changes when viewed obliquely. This is because light bends on travelling from one medium to another. The apparent depth of the tank when viewed obliquely is less than the near-normal viewing.

(e) Yes

The refractive index of diamond (2.42) is more than that of ordinary glass (1.5). The critical angle for diamond is less than that for glass. A diamond cutter uses a large angle of incidence to ensure that the light entering the diamond is totally reflected from its faces. This is the reason for the sparkling effect of a diamond.

19)

Focal length of the convex lens,  $f_1 = 30$  cm

Focal length of the concave lens,  $f_2 = -20$  cm

Distance between the two lenses,  $d = 8.0$  cm

(a) When the parallel beam of light is incident on the convex lens first:

According to the lens formula, we have:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

Where,

$u_1 =$  Object distance  $= \infty$

$v_1 =$  Image distance

$$\frac{1}{v_1} = \frac{1}{30} - \frac{1}{\infty} = \frac{1}{30}$$

$\therefore v_1 = 30$  cm

The image will act as a virtual object for the concave lens.

Applying lens formula to the concave lens, we have:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

Where,

$u_2 =$  Object distance

$= (30 - d) = 30 - 8 = 22$  cm

$v_2 =$  Image distance

$$\frac{1}{v_2} = \frac{1}{22} - \frac{1}{20} = \frac{10-11}{220} = \frac{-1}{220}$$

$\therefore v_2 = -220$  cm

The parallel incident beam appears to diverge from a point that is  $\left(220 - \frac{d}{2} = 220 - 4\right) 216$  cm from the centre of

the combination of the two lenses.

(ii) When the parallel beam of light is incident, from the left, on the concave lens first:

According to the lens formula, we have:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} \quad \text{Where,}$$

$$u_2 = \text{Object distance} = -\infty$$

$$v_2 = \text{Image distance}$$

$$\frac{1}{v_2} = \frac{1}{-20} + \frac{1}{-\infty} = -\frac{1}{20}$$

$$\therefore v_2 = -20 \text{ cm}$$

The image will act as a real object for the convex lens.

Applying lens formula to the convex lens, we have:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

Where,

$$u_1 = \text{Object distance}$$

$$= -(20 + d) = -(20 + 8) = -28 \text{ cm}$$

$$v_1 = \text{Image distance}$$

$$\frac{1}{v_1} = \frac{1}{30} + \frac{1}{-28} = \frac{14-15}{420} = \frac{-1}{420}$$

$$\therefore v_1 = -420 \text{ cm}$$

Hence, the parallel incident beam appear to diverge from a point that is  $(420 - 4) 416 \text{ cm}$  from the left of the centre of the combination of the two lenses

The answer does depend on the side of the combination at which the parallel beam of light is incident. The notion of effective focal length does not seem to be useful for this combination.

(b) Height of the image,  $h_1 = 1.5 \text{ cm}$

Object distance from the side of the convex lens,  $u_1 = -40 \text{ cm}$

$$|u_1| = 40 \text{ cm}$$

According to the lens formula:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

Where,

$$v_1 = \text{Image distance}$$

$$\frac{1}{v_1} = \frac{1}{30} + \frac{1}{-40} = \frac{4-3}{120} = \frac{1}{120}$$

$$\therefore v_1 = 120 \text{ cm}$$

$$m = \frac{v_1}{|u_1|}$$

$$= \frac{120}{40} = 3$$

Hence, the magnification due to the convex lens is 3.

The image formed by the convex lens acts as an object for the concave lens.

According to the lens formula:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

Where,

$$u_2 = \text{Object distance}$$

$$= +(120 - 8) = 112 \text{ cm.}$$

$$v_2 = \text{Image distance}$$

$$\frac{1}{v_2} = \frac{1}{-20} + \frac{1}{112} = \frac{-112+20}{2240} = \frac{-92}{2240}$$

$$\therefore v_2 = \frac{2240}{92} \text{ cm}$$

Magnification,

$$m' = \left| \frac{v_2}{u_2} \right|$$

$$= \frac{2240}{92} \times \frac{1}{112} = \frac{20}{92}$$

Hence, the magnification due to the concave lens is  $\frac{20}{92}$

The magnification produced by the combination of the two lenses is calculated as:

$m \times m'$

$$= 3 \times \frac{20}{92} = \frac{60}{92} = 0.652$$

The magnification of the combination is given as:

$$\frac{h_2}{h_1} = 0.652$$

$$h_2 = 0.652 \times h_1$$

Where,

$h_1$  = Object size = 1.5 cm

$h_2$  = Size of the image

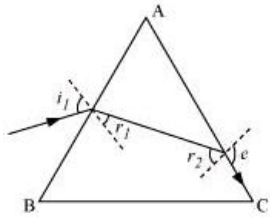
$$\therefore h_2 = 0.652 \times 1.5 = 0.98 \text{ cm}$$

Hence, the height of the image is 0.98 cm

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- 20) The incident, refracted, and emergent rays associated with a glass prism ABC are shown in the given figure.



Angle of prism,  $\angle A = 60^\circ$

Refractive index of the prism,  $\mu = 1.524$

$i_1$  = Incident angle

$r_1$  = Refracted angle

$r_2$  = Angle of incidence at the face AC

$e$  = Emergent angle =  $90^\circ$

According to Snell's law, for face AC, we can have:

$$\frac{\sin e}{\sin r_2} = \mu$$

$$\sin r_2 = \frac{1}{\mu} \times \sin 90^\circ$$

$$= \frac{1}{1.524} = 0.6562$$

$$\therefore r_2 = \sin^{-1} 0.6562 \approx 41^\circ$$

It is clear from the figure that angle  $A = r_1 + r_2$

$$\therefore r_1 = A - r_2 = 60 - 41 = 19^\circ$$

According to Snell's law, we have the relation:

$$\mu = \frac{\sin i_1}{\sin r_1}$$

$$\sin i_1 = \mu \sin r_1$$

$$= 1.524 \times \sin 19^\circ = 0.496$$

$$\therefore i_1 = 29.75^\circ$$

Hence, the angle of incidence is  $29.75^\circ$ .

- 21)

Note: Here we took focal Length as 10 cm because if we take it as 9 cm then image distance will be zero, which does not make any sense.

(a) Area of each square,  $A = 1 \text{ mm}^2$

Object distance,  $u = -9 \text{ cm}$

Focal length of a converging lens,  $f = 9 \text{ cm}$

For image distance  $v$ , the lens formula can be written as:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} + \frac{1}{9}$$

$$\frac{1}{v} = \frac{1}{90}$$

$$\therefore v = -90 \text{ cm}$$

$$\text{Magnification, } m = \frac{v}{u}$$

$$= \frac{-90}{-9} = 10$$

$$\therefore \text{Area of each square in the virtual image} = (10)^2 A$$

$$= 10^2 \times 1 = 100 \text{ mm}^2$$

$$= 1 \text{ cm}^2$$

$$(b) \text{ Magnifying power of the lens} = \frac{d}{|\mu|} = \frac{25}{9} = 2.8$$

(c) The magnification in (a) is not the same as the magnifying power in (b).

The magnification magnitude is  $\left( \left| \frac{v}{u} \right| \right)$  and the magnifying power is  $\left( \frac{d}{|u|} \right)$

The two quantities will be equal when the image is formed at the near point (25 cm).

22) (a) The maximum possible magnification is obtained when the image is formed at the near point ( $d = 25 \text{ cm}$ ).

Image distance,  $v = -d = -25 \text{ cm}$

Focal length,  $f = 10 \text{ cm}$

Object distance =  $u$

According to the lens formula, we have:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{-25} - \frac{1}{10} = \frac{-2-5}{50} = -\frac{7}{50}$$

$$\therefore u = -\frac{50}{7} = -7.14 \text{ cm}$$

Hence, to view the squares distinctly, the lens should be kept 7.14 cm away from them

$$(b) \text{ Magnification} = \left| \frac{v}{u} \right| = \frac{25}{\frac{50}{7}} = 3.5$$

$$(c) \text{ Magnifying power} = \frac{d}{\frac{50}{7}} = 3.5$$

Since the image is formed at the near point (25 cm), the magnifying power is equal to the magnitude of magnification.

23)

Area of the virtual image of each square,  $A = 6.25 \text{ mm}^2$

Area of each square,  $A_0 = 1 \text{ mm}^2$

Hence, the linear magnification of the object can be calculated as:

$$m = \sqrt{\frac{A}{A_0}}$$

$$\sqrt{\frac{6.25}{1}} = 2.5$$

$$\text{But } m = \frac{\text{Image distance } (v)}{\text{Object distance } (u)}$$

$$\therefore v = mu$$

$$= 2.5 u \dots(1)$$

Focal length of the magnifying glass,  $f = 10 \text{ cm}$

According to the lens formula, we have the relation:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{2.5u} - \frac{1}{u} = \frac{1}{u} \left( \frac{1}{2.5} - 1 \right) = \frac{1}{u} \left( \frac{1-2.5}{2.5} \right)$$

$$\therefore u = - \frac{1.5 \times 10}{2.5} = -6$$

And  $v = 2.5u$

$$= 2.5 \times 6 = -15 \text{ cm}$$

The virtual image is formed at a distance of 15 cm, which is less than the near point (i.e., 25 cm) of a normal eye. Hence, it cannot be seen by the eyes distinctly.

24)

(a) Though the image size is bigger than the object, the angular size of the image is equal to the angular size of the object. A magnifying glass helps one see the objects placed closer than the least distance of distinct vision (i.e., 25 cm). A closer object causes a larger angular size. A magnifying glass provides angular magnification. Without magnification, the object cannot be placed closer to the eye. With magnification, the object can be placed much closer to the eye.

(b) Yes, the angular magnification changes. When the distance between the eye and a magnifying glass is increased, the angular magnification decreases a little. This is because the angle subtended at the eye is slightly less than the angle subtended at the lens. Image distance does not have any effect on angular magnification.

(c) The focal length of a convex lens cannot be decreased by a greater amount. This is because making lenses having very small focal lengths is not easy. Spherical and chromatic aberrations are produced by a convex lens having a very small focal length.

(d) The angular magnification produced by the eyepiece of a compound microscope is  $\left[ \left( \frac{25}{f_e} \right) + 1 \right]$

Where,

$f_e$  = Focal length of the eyepiece

It can be inferred that if  $f_e$  is small, then angular magnification of the eyepiece will be large.

The angular magnification of the objective lens of a compound microscope is given as  $\frac{1}{(u_o/f_o)}$

Where,

$u_o$  = Object distance for the objective lens

$f_o$  = Focal length of the objective

The magnification is large when  $u_o > f_o$ . In the case of a microscope, the object is kept close to the objective lens. Hence, the object distance is very little. Since  $u_o$  is small,  $f_o$  will be even smaller. Therefore,  $f_e$  and  $f_o$  are both small in the given condition.

(e) When we place our eyes too close to the eyepiece of a compound microscope, we are unable to collect much refracted light. As a result, the field of view decreases substantially. Hence, the clarity of the image gets blurred.

The best position of the eye for viewing through a compound microscope is at the eye-ring attached to the eyepiece. The precise location of the eye depends on the separation between the objective lens and the eyepiece.

- 25) Focal length of the objective lens,  $f_o = 1.25$  cm  
 Focal length of the eyepiece,  $f_e = 5$  cm  
 Least distance of distinct vision,  $d = 25$  cm  
 Angular magnification of the compound microscope = 30X  
 Total magnifying power of the compound microscope,  $m = 30$   
 The angular magnification of the eyepiece is given by the relation:

$$m_e = \left(1 + \frac{d}{f_e}\right)$$

$$= \left(1 + \frac{25}{5}\right) = 6$$

The angular magnification of the objective lens ( $m_o$ ) is related to  $m_e$  as:

$$m_o m_e = m$$

$$m_o = \frac{m}{m_e}$$

$$= \frac{30}{6} = 5$$

We also have the relation:

$$m = \frac{\text{Image distance for the objective lens } (v_o)}{\text{Object distance for the objective lens } (u_o)}$$

$$5 = \frac{v_o}{-u_o}$$

$$\therefore v_o = -5u_o \dots\dots\dots(1)$$

Applying the lens formula for the objective lens:

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\frac{1}{1.25} = \frac{1}{-5u_o} - \frac{1}{u_o} = \frac{-6}{5u_o}$$

$$\therefore u_o = \frac{-6}{5} \times 1.25 = -1.5 \text{ cm}$$

$$\text{And } v_o = -5u_o$$

$$= -5 \times (-1.5) = 7.5 \text{ cm}$$

The object should be placed 1.5 cm away from the objective lens to obtain the desired magnification.

Applying the lens formula for the eyepiece:

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

Where,

$$v_e = \text{Image distance for the eyepiece} = -d = -25 \text{ cm}$$

$$u_e = \text{Object distance for the eyepiece}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= \frac{-1}{25} - \frac{1}{5} = -\frac{6}{25}$$

$$\therefore u_e = -4.17 \text{ cm}$$

$$\text{Separation between the objective lens and the eyepiece} = |u_e| + |v_o|$$

$$= 4.17 + 7.5$$

$$= 11.67 \text{ cm}$$

Therefore, the separation between the objective lens and the eyepiece should be 11.67 cm.

26) Focal length of the objective lens,  $f_o = 140$  cm

Focal length of the eyepiece,  $f_e = 5$  cm

Least distance of distinct vision,  $d = 25$  cm

(a) When the telescope is in normal adjustment, its magnifying power is given as:

$$m = \frac{f_o}{f_e}$$

$$= \frac{140}{5} = 28$$

(b) When the final image is formed at  $d$ , the magnifying power of the telescope is given as:

$$\frac{f_o}{f_e} \left[ 1 + \frac{f_e}{d} \right]$$

$$= \frac{140}{5} \left[ 1 + \frac{2}{25} \right]$$

$$= 28 [1 + 0.2]$$

$$= 28 \times 1.2 = 33.6$$

27) Focal length of the objective lens,  $f_o = 140$  cm

Focal length of the eyepiece,  $f_e = 5$  cm

(a) In normal adjustment, the separation between the objective lens and the eyepiece =  $f_o + f_e = 140 + 5 = 145$  cm

(b) Height of the tower,  $h_1 = 100$  m

Distance of the tower (object) from the telescope,  $u = 3$  km = 3000 m

The angle subtended by the tower at the telescope is given as:

$$\theta = \frac{h_1}{u}$$

$$= \frac{100}{3000} = \frac{1}{30} \text{ rad}$$

The angle subtended by the image produced by the objective lens is given as:

$$\theta = \frac{h_2}{f_o} = \frac{h_2}{140} \text{ rad}$$

Where,

$h_2$  = Height of the image of the tower formed by the objective lens

$$\frac{1}{30} = \frac{h_2}{140}$$

$$\therefore h_2 = \frac{140}{30} = 4.7 \text{ cm}$$

Therefore, the objective lens forms a 4.7 cm tall image of the tower.

(c) Image is formed at a distance,  $d = 25$  cm

The magnification of the eyepiece is given by the relation:

$$m = 1 + \frac{d}{f_e}$$

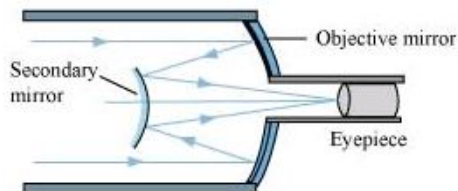
$$= 1 + \frac{25}{5} = 1 + 5 = 6$$

Height of the final image =  $mh_2 = 6 \times 4.7 = 28.2$  cm

Hence, the height of the final image of the tower is 28.2 cm.

28)

The following figure shows a Cassegrain telescope consisting of a concave mirror and a convex mirror.



Distance between the objective mirror and the secondary mirror,  $d = 20$  mm

Radius of curvature of the objective mirror,  $R_1 = 220$  mm

Hence, focal length of the objective mirror,  $f_1 = \frac{R_1}{2} = 110$

Radius of curvature of the secondary mirror,  $R_2 = 140$  mm

Hence, focal length of the secondary mirror,  $f_2 = \frac{R_2}{2} = \frac{140}{2} = 70$  mm

The image of an object placed at infinity, formed by the objective mirror, will act as a virtual object for the secondary mirror.

Hence, the virtual object distance for the secondary mirror,  $u = f_1 - d$

$$= 110 - 20$$

$$= 90 \text{ mm}$$

Applying the mirror formula for the secondary mirror, we can calculate image distance ( $v$ ) as:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_2}$$

$$\frac{1}{v} = \frac{1}{f_2} - \frac{1}{u}$$

$$\frac{1}{70} - \frac{1}{90} = \frac{9-7}{630} = \frac{2}{630}$$

$$\therefore v = \frac{630}{2} = 315 \text{ mm}$$

Hence, the final image will be formed 315 mm away from the secondary mirror.

29)

Focal length of the convex lens,  $f_1 = 30$  cm

The liquid acts as a mirror. Focal length of the liquid =  $f_2$

Focal length of the system (convex lens + liquid),  $f = 45$  cm

For a pair of optical systems placed in contact, the equivalent focal length is given as:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$$

$$= \frac{1}{45} - \frac{1}{30} = -\frac{1}{90}$$

$$\therefore f_2 = -90 \text{ cm}$$

Let the refractive index of the lens be  $\mu_1$  and the radius of curvature of one surface be  $R$ . Hence, the radius of curvature of the other surface is  $-R$

$R$  can be obtained using the relation:

$$\frac{1}{f_1} = (\mu_1 - 1) \left( \frac{1}{R} + \frac{1}{-R} \right)$$

$$\frac{1}{30} = (1.5 - 1) \left( \frac{2}{R} \right)$$

$$\therefore R = \frac{30}{0.5 \times 2} = 30 \text{ cm}$$

Let  $\mu_2$  be the refractive index of the liquid.

Radius of curvature of the liquid on the side of the plane mirror =  $\infty$

Radius of curvature of the liquid on the side of the lens,  $R = -30$  cm

The value of  $\mu_2$  can be calculated using the relation:

$$\frac{1}{f_2} = (\mu_2 - 1) \left[ \frac{1}{-R} - \frac{1}{\infty} \right]$$

$$\frac{-1}{90} = (\mu_2 - 1) \left[ \frac{1}{+30} - 0 \right]$$

$$\mu_2 - 1 = \frac{1}{3}$$

$$\therefore \mu_2 = \frac{4}{3} = 1.33$$

Hence, the refractive index of the liquid is 1.33.



30)

Least distance of distinct vision,  $d = 25 \text{ cm}$

Far point of a normal eye,  $d' = \infty$

Converging power of the cornea,  $P_c = 40 \text{ D}$

Least converging power of the eye-lens,  $P_e = 20 \text{ D}$

To see the objects at infinity, the eye uses its least converging power.

Power of the eye-lens,  $P = P_c + P_e = 40 + 20 = 60 \text{ D}$

Power of the eye-lens is given as:  $P = \frac{1}{\text{Focal length of the eye lens}(f)}$

$$\begin{aligned} f &= \frac{1}{P} \\ &= \frac{1}{60} \text{ D} \\ &= \frac{100}{60} = \frac{5}{3} \text{ cm} \end{aligned}$$

To focus an object at the near point, object distance ( $u$ ) =  $-d = -25 \text{ cm}$

Focal length of the eye-lens = Distance between the cornea and the retina = Image distance

Hence, image distance,  $v = \frac{5}{3} \text{ cm}$

According to the lens formula, we can write:

Where,

$f'$  = Focal length

$$\frac{1}{f'} = \frac{1}{v} + \frac{1}{u} = \frac{15+1}{25} = \frac{16}{25} \text{ cm}$$

Power,  $P' = \frac{1}{f'}$ ,  $\times 100$

$$= \frac{16}{25} \times 100 = 64 \text{ D}$$

$\therefore$  Power of the eye-lens =  $64 - 40 = 24 \text{ D}$

Hence, the range of accommodation of the eye-lens is from  $20 \text{ D}$  to  $24 \text{ D}$ .

31) (a) Focal length of the magnifying glass,  $f = 5$  cm

Least distance of distance vision,  $d = 25$  cm

Closest object distance =  $u$

Image distance,  $v = -d = -25$  cm

According to the lens formula, we have:

$$\begin{aligned}\frac{1}{f} &= \frac{1}{v} - \frac{1}{u} \\ \frac{1}{5} &= \frac{1}{-25} - \frac{1}{u} \\ \frac{1}{u} &= \frac{1}{-25} - \frac{1}{5} = \frac{-5-1}{25} = -\frac{6}{25} \\ \therefore u &= -\frac{25}{6} = -4.167 \text{ cm}\end{aligned}$$

Hence, the closest distance at which the person can read the book is 4.167 cm.

For the object at the farthest distant ( $u'$ ), the image distance ( $v'$ ) =  $\infty$

According to the lens formula, we have:

$$\begin{aligned}\frac{1}{f} &= \frac{1}{v'} - \frac{1}{u'} \\ \frac{1}{5} &= \frac{1}{\infty} - \frac{1}{u'} = -\frac{1}{u'} \\ \therefore u' &= -5 \text{ cm}\end{aligned}$$

Hence, the farthest distance at which the person can read the book is 5 cm.

(b) Maximum angular magnification is given by the relation:

$$\begin{aligned}\alpha_{\max} &= \frac{d}{|u|} \\ &= \frac{25}{\frac{25}{6}} = 6\end{aligned}$$

Minimum angular magnification is given by the relation:

$$\begin{aligned}\alpha_{\min} &= \frac{d}{|u'|} \\ &= \frac{25}{5} = 5\end{aligned}$$

32)

Let a spherical surface separate a rarer medium of refractive index  $n_1$  from the second medium of refractive index  $n_2$ . Let  $C$  be the centre of curvature and  $R = MC$  be the radius of the surface.

Consider a point object  $O$  lying on the principal axis of the surface. Let a ray starting from  $O$  incident normally on the surface along  $OM$  and pass straight. Let another ray of light incident on  $NM$  along  $ON$  and refract along  $NI$ . From  $M$  draw  $MN$  perpendicular to  $OI$ .

The above figure shows the geometry of the formation of image  $I$  of an object  $O$  and the principal axis of a spherical surface with centre of curvature  $C$  and radius of curvature  $R$ .

Here, we have to make following assumptions,

(i) the aperture of the surface is small as compared to the other distance involved.

(ii)  $NM$  will be taken as nearly equal to the length of the perpendicular from the point  $N$  on the principal axis.

$$\tan \angle NOM = \frac{MN}{OM}, \quad \tan \angle NCM = \frac{MN}{MC}$$

$$\tan \angle NIM = \frac{MN}{MI}$$

For  $\triangle NOC$ , is the exterior angle.

$$\therefore \angle i = \angle NOM + \angle NCM$$

$$\text{For small angles, } i = \frac{MN}{OM} + \frac{MN}{NC} \dots\dots(i)$$

Similarly,  $r = \angle NCM - \angle NIM$

$$\Rightarrow r = \frac{MN}{NC} - \frac{MN}{NI} \dots\dots(ii)$$

By Snell's law, we get

$$n_1 \sin i = n_2 \sin r$$

For small angles,  $n_1 i = n_2 r$

Put the values of  $i$  and  $r$  from Eqs. (i) and (ii), we get

$$n_1 \left( \frac{MN}{OM} + \frac{MN}{MC} \right) = n_2 \left( \frac{MN}{MC} - \frac{MN}{MI} \right)$$

$$\Rightarrow \frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \dots\dots(iii)$$

Applying new cartesian sign conventions, we get

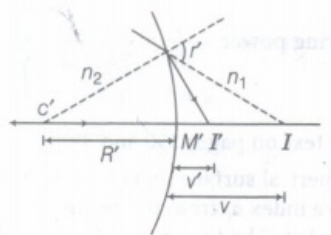
$$OM = -u, \quad MI = +v$$

$$\text{and } MC = +R$$

Substituting this in Eq. (iii), we get

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \dots\dots(iv)$$

Now, the image  $I'$  acts as a virtual object for the second surface that will form a real at  $I$ . As, refraction takes place from denser to rarer medium,



$$\therefore \frac{-n_2}{v} + \frac{n_1}{v'} = \frac{n_2 - n_1}{-R} \dots\dots(v)$$

On adding Eqs. (iv) and (v), we get

$$\frac{1}{f} = (n_{21} - 1) \left( \frac{1}{R} - \frac{1}{R'} \right) \left[ \because n_{21} = \frac{n_2}{n_1}, \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \right]$$

33) Since,  $\angle A = 60^\circ$

$$\begin{aligned} \mu &= \frac{\sin\left(\frac{60^\circ + 60^\circ}{2}\right)}{\sin(60^\circ/2)} \\ &= \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}}{2} \times \frac{2}{1} \\ &= \sqrt{3} = 1.732 \end{aligned}$$

34)

The magnifying power of a telescope is equal to the ratio of the visual angle subtended at the eye by final image formed at least distance' of distinct vision to the visual angle subtended at naked eye by the object at infinity.

$$\text{Magnification, } m = \frac{I}{O} = \frac{v_0}{u_0} = \frac{f_0}{u_0}$$

$$\Rightarrow \frac{I}{100} = \frac{150 \times 10^{-2}}{3 \times 10^3}$$

$$\Rightarrow I = 5 \times 10^{-2} \text{ m} = 5 \text{ cm}$$

35)

(ii) Given, magnification,  $m = 20$

Magnification of eyepiece,  $m_e = 5$

Least distance vision,  $D = 20 \text{ cm}$

Distance between the object and eyepiece,

$L = 14 \text{ cm}$

We know that, magnification,  $m = m_e \times m_o$

$$\Rightarrow m_o = \frac{m}{m_e} = \frac{20}{5} = 4$$

$$\text{As, } m_e = 1 + \frac{D}{f_e}$$

where,  $f_e$  is focal length of eyepiece.

$$\Rightarrow 5 = 1 + \frac{20}{f_e} \Rightarrow f_e = 5 \text{ cm}$$

Using lens formula for eyepiece,

$$\frac{1}{u_e} = \frac{-1}{20} - \frac{1}{5} = \frac{-5}{20} = \frac{-1}{4}$$

$\Rightarrow u_e = -4 \text{ cm}$  (object distance for eyepiece)

$$\Rightarrow L = v_o + |u_e|$$

$$\Rightarrow v_o = L - |u_e|$$

$$\Rightarrow v_o = L - |u_e|$$

$$= 14 - 4 = 10 \text{ cm}$$

Magnification produced by object,  $m_o = -\frac{v_o}{u_o}$

Object distance for object,

$$u_o = \frac{-v_o}{m_o} = \frac{-10}{4} = -2.5 \text{ cm}$$

Using lens formula for object,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{10} - \frac{1}{-2.5} = \frac{1}{10} + \frac{1}{2.5}$$

$$f_o = 2 \text{ cm}$$

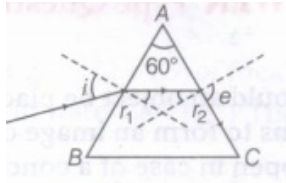
- 36) (b) Power of a lens increases if red light is replaced by violet light because

$$P = \frac{1}{f} = (a\mu_g - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

As refractive index is maximum for violet light in visible region of spectrum.

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- 37) (ii) Given, the emergent ray grazes along the face AC,



$$e = 90^\circ$$

$$\mu = \sqrt{2}$$

$$\frac{\sin i}{\sin r_1} = \mu = \frac{\sin e}{\sin r_2}$$

$$\Rightarrow \frac{\sin 90^\circ}{\sin r_2} = \sqrt{2}$$

$$\text{i.e. } \sin r_2 = \frac{1}{\sqrt{2}} \text{ or } r_2 = 45^\circ$$

$$\Rightarrow r_1 + r_2 = \angle A = 60^\circ$$

$$r_1 = 60 - r_2 = 15^\circ$$

$$\Rightarrow \frac{\sin i}{\sin 15^\circ} = \sqrt{2}$$

$$\Rightarrow i = 21.47^\circ$$

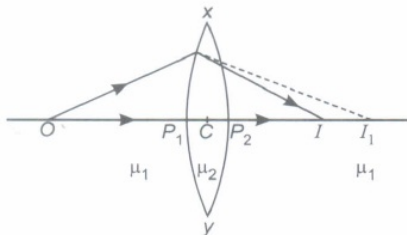

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- 38)

Lens Maker's Formula: Consider a thin convex lens made of a material of absolute refractive index  $\mu_2$  placed in a rarer medium of absolute refractive index  $\mu_1$ . Let  $\mu$  be the refractive index of the material of the lens with respect to the medium surrounding it.

$$\text{So } \mu = \frac{\mu_2}{\mu_1}$$

Let us first consider the refraction of light from the object O at surface  $XP_1Y$  of the lens of radius of curvature  $R_1$ . Let  $I_1$  be the real image formed due to refraction at surface  $XP_1Y$  assuming that the material of the lens extends beyond  $I_1$  then



$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad \dots\dots(i)$$

Here  $v_1 = P_1I_1 = CI_1$  and  $u = -OP_1 = -OC$

$$\therefore \frac{\mu_2}{CI_1} + \frac{\mu_1}{OC} = \frac{\mu_2 - \mu_1}{R_1} \quad \dots\dots(ii)$$

Let us now consider the refraction at surface  $XP_2Y$  of the lens of radius of curvature  $R_2$ . For this surface  $I_1$  acts as the virtual object. Therefore, the final image (real) of the object O is formed at I as shown in the diagram above.

For refraction at surface,  $XP_2Y$ , we have

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \quad \dots\dots(iii)$$

Here  $v = P_2I = CI$  and  $v_1 = P_2I_1 = CI_1$

Therefore, equation (iii) can be rewritten as

$$\frac{\mu_1}{CI} - \frac{\mu_2}{CI_1} = \frac{\mu_1 - \mu_2}{R_2} \quad \dots\dots(iv)$$

From equations (ii) and (iv), on adding, we get

$$\frac{\mu_1}{CI} - \frac{\mu_2}{CI_1} + \frac{\mu_2}{CI_1} + \frac{\mu_1}{OC} = (\mu_2 - \mu_1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\text{or } \mu_1 \left[ \frac{1}{CI} + \frac{1}{OC} \right] = (\mu_2 - \mu_1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

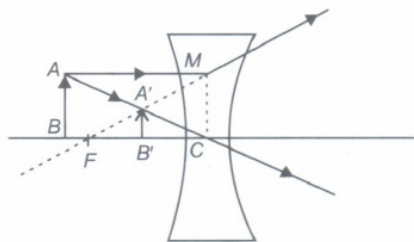
$$\text{or } \frac{1}{CI} + \frac{1}{OC} = \left( \frac{\mu_2}{\mu_1} - 1 \right) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

As  $CI = v$  and  $OC = -u$  and  $\frac{\mu_2}{\mu_1} = \mu$

$$\therefore \frac{1}{v} - \frac{1}{u} = (\mu - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\frac{1}{f} = (\mu - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

To find the rough focal length of a converging lens, we put a screen in front of it and make it to a distance for which a clear image of a distant object is formed on the screen. The distance, thus, measured is the rough focal length of the lens.



The image formed is virtual, erect and smaller in size.

As  $\Delta A'B'C \sim \Delta ABC$

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \quad \dots\dots\dots(i)$$

Again,  $\Delta A'B'F \sim \Delta CMF$

$$\frac{A'B'}{CM} = \frac{B'F}{CF}$$

$$\frac{A'B'}{AB} = \frac{B'F}{CF} \quad (\because CM = AB) \quad \dots\dots\dots(ii)$$

From equations (i) and (ii), we get

$$\frac{CB'}{CB} = \frac{B'F}{CF} = \frac{CF - CB'}{CF}$$

Using new cartesian sign conventions, let

$$CB' = -v, CB = -u, CF = -f$$

A graph between  $\frac{1}{u}$  and  $\frac{1}{v}$

$$\text{Thus, } \frac{-v}{-u} = \frac{-f+v}{-f} \Rightarrow fv = uf - uv$$

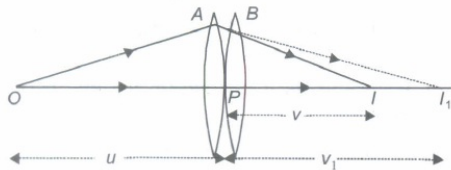
$$\Rightarrow f(u - v) = uv \Rightarrow \frac{u-v}{uv} = \frac{1}{f}$$

$$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

40)

(a)

Power of a lens is the measure of convergence or divergence which a lens can introduce in beam of light falling on it. The SI unit of power is dioptre (D).



For the 1<sup>st</sup> lens, we have relation

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u} \quad \dots\dots(i)$$

For the 2<sup>nd</sup> lens, the relation is

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1} \quad \dots\dots(ii)$$

Here an image formed by the 1<sup>st</sup> lens acts as a virtual object for the 2<sup>nd</sup> lens.

Adding equations (i) and (ii), we get

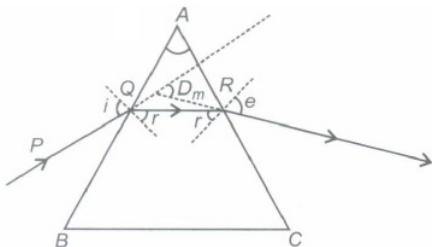
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

If this two lens system is considered as an equivalent single lens of focal length  $f$ , we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \text{Power } P = \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

(b) Given: Angle of prism,  $A = 60^\circ$ ,



Refractive index,  $n = \sqrt{3}$

In this case refracted ray is going parallel to base, hence it is the case of minimum deviation. We know

$$r_1 + r_2 = A$$

$$i + e = A + D \quad \dots\dots(i)$$

For minimum deviation  $r_1 = r_2$ ,  $i = e$  and  $D = D_m$

$$\therefore 2r = A \Rightarrow r = \frac{60}{2} = 30^\circ$$

$$\therefore \frac{\sin i}{\sin r} = \sqrt{3}$$

$$\Rightarrow \sin i = \frac{\sqrt{3}}{2} \Rightarrow i = 60^\circ$$

$$\text{From equation (i) } 2i = A + D_m \Rightarrow D_m = 60^\circ$$

41)



(a) The ray diagram showing the passage of a ray of light through a prism of refracting angle A.

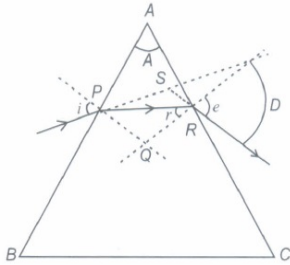
Here  $\angle BAC = \angle A$ ,  $\angle SPQ = \angle i$ ,  $\angle SRQ = \angle e$

In trapezium APQR,

$$\angle A + \angle PQR = 180^\circ \dots\dots(i)$$

In  $\Delta PQR$ ,

$$\angle r + \angle PQR + \angle r' = 180^\circ \dots\dots(ii)$$



In trapezium PQRS,

$$\angle SPQ + \angle PQR + \angle SRQ + \angle PSR = 360^\circ$$

$$\angle i + (180^\circ - \angle A) + \angle e + \angle PSR = 360^\circ$$

$$\text{Also } \angle PSR + \angle D_{\min} = 180^\circ$$

$$\therefore \angle i + (180^\circ - \angle A) + \angle e + 180^\circ - \angle D_{\min} = 360^\circ$$

$$\text{or } \angle i + \angle e = \angle A + \angle D_{\min} \dots\dots(iii)$$

From equations (i) and (ii), we get

$$\angle r + \angle r' = \angle A$$

$$\text{when } \angle r = \angle r', 2\angle r = \angle A$$

$$\text{or } \angle r = \frac{\angle A}{2} \text{ and } \angle i = \angle e$$

$$\therefore \text{Equation (iii) becomes } 2\angle i = \angle A + \angle D_{\min}$$

$$\Rightarrow \angle i = \frac{\angle A + \angle D_{\min}}{2}$$

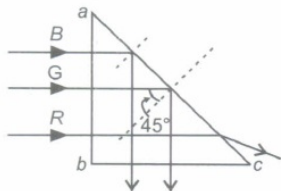
$$\therefore \mu = \frac{\sin i}{\sin r}$$

$$\mu = \frac{\sin\left(\frac{A + D_{\min}}{2}\right)}{\sin\frac{A}{2}}$$

(b) For any ray of light incident normally on the face ab of the prism abc, the value of critical angle is  $45^\circ$ , the angle of refraction would be of  $90^\circ$  and the refractive index of the material of prism is

$${}_g\mu_a = \frac{\sin 45^\circ}{\sin 90^\circ}$$

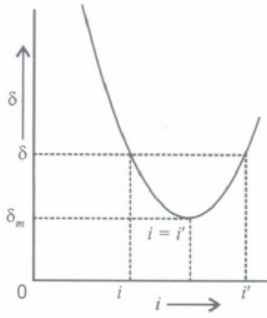
$$\Rightarrow a_{\mu_g} = \sqrt{2} = 1.414$$



Thus, for greater values to refractive indices, an internal reflection will take place, i.e. green and blue colours will get internally reflected and red colour will emerge out of the prism.

42)

(i)



Expression for refractive index of the prism:

(a) The ray diagram showing the passage of a ray of light through a prism of refracting angle A.

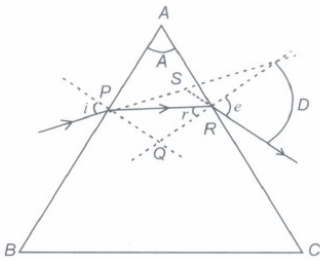
Here  $\angle BAC = \angle A$ ,  $\angle SPQ = \angle i$ ,  $\angle SRQ = \angle e$

In trapezium APQR,

$$\angle A + \angle PQR = 180^\circ \dots\dots(i)$$

In  $\Delta PQR$ ,

$$\angle r + \angle PQR + \angle r' = 180^\circ \dots\dots(ii)$$



In trapezium PQRS,

$$\angle SPQ + \angle PQR + \angle SRQ + \angle PSR = 360^\circ$$

$$\angle i + (180^\circ - \angle A) + \angle e + \angle PSR = 360^\circ$$

$$\text{Also } \angle PSR + \angle D_{\min} = 180^\circ$$

$$\therefore \angle i + (180^\circ - \angle A) + \angle e + 180^\circ - \angle D_{\min} = 360^\circ$$

$$\text{or } \angle i + \angle e = \angle A + \angle D_{\min} \dots\dots(iii)$$

From equations (i) and (ii), we get

$$\angle r + \angle r' = \angle A$$

$$\text{when } \angle r = \angle r', 2\angle r = \angle A$$

$$\text{or } \angle r = \frac{\angle A}{2} \text{ and } \angle i = \angle e$$

$$\therefore \text{Equation (iii) becomes } 2\angle i = \angle A + \angle D_{\min}$$

$$\Rightarrow \angle i = \frac{\angle A + \angle D_{\min}}{2}$$

$$\therefore \mu = \frac{\sin i}{\sin r}$$

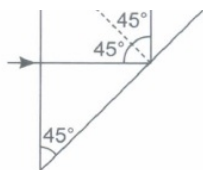
$$\mu = \frac{\sin \left( \frac{A + D_{\min}}{2} \right)}{\sin \frac{A}{2}}$$

(ii) The splitting of white light into its constituent colours when it passes through a dispersive medium is called dispersion of light.

Cause: Different colours travel with different speeds in denser medium.

(iii) At  $i = 45^\circ$  the ray of light is totally reflected. Therefore, the minimum value of refractive index of glass.



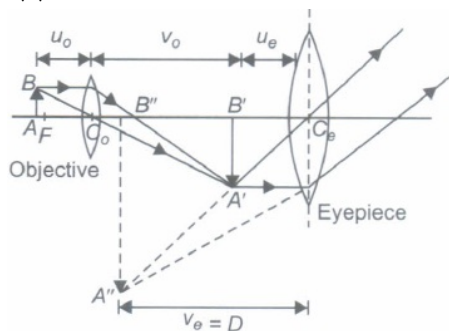


$$n = \frac{1}{\sin i_c} = \frac{1}{\sin 45^\circ}$$

$$n = \sqrt{2} = 1.41$$

43)

(a)



Limit of resolution: The minimum linear or angular separation between two point objects at which they can be just separately seen or resolved by an optical instrument is called limit of resolution.

The limit of resolution depends on:

- (i) wavelength of light used, and
- (ii) medium between object and objective lens.

The resolving power of microscope is the reciprocal of its limit of resolution.

(b) The resolving power of a microscope can be increased by

- (i) decreasing wavelength, and
- (ii) increasing refractive index of the medium between object and objective of the microscope .

(c) A telescope produces an (angularly) magnified image of the far object and thereby enables us to resolve them.

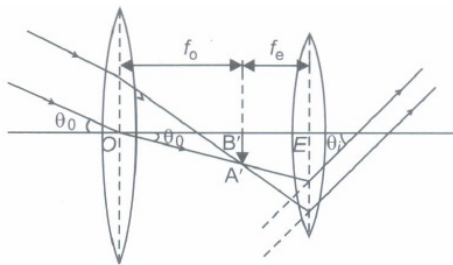
A microscope magnifies small objects which are near to our eyes.

44)

A compound microscope consists of two convex lenses. The lens facing the object to be seen is called an objective lens. The lens facing the eye is called an eye lens. Generally, the eye lens is a combination of lenses and is called an eyepiece. The aperture and the focal length of the objective lens are smaller as compared with those of eye lens. The object to be magnified is placed just beyond the focal point of the objective lens which forms its real, magnified and inverted image. This image acts as the object for the eye lens whose position is so adjusted that the final image formed by the eye lens is at the distance of distinct vision as shown in the diagram.

- (b) (i) To achieve a large magnification of small object, the eyepiece and the objective must have short focal lengths.
- (ii) If we place our eyes too close to the eyepiece, the area of the pupil of the eye is less than the area of the eye-ring. So, our eyes will not collect much of the light and our field of view will get reduced.

45)



### ray diagram of a refracting telescope

Magnifying power is the ratio of angle subtended at the eye by the final image to the angle subtended by the object at the eye.

$$m = \frac{f_o}{f_e}$$

We can increase the magnifying power by:

- (i) increasing focal length of an objective.
- (ii) decreasing focal length of an eyepiece.

Limitations:

- (i) Refracting type telescope suffers from chromatic aberration.
- (ii) They have small resolving power.

Advantages of reflecting type telescope

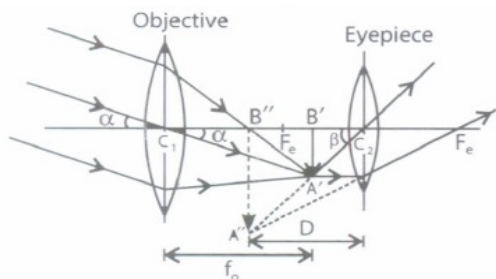
- (i) It is free from chromatic aberration as there is no refraction.
- (ii) The aperture of mirror can be kept larger in comparison to the aperture of lenses, because their grinding/polishing is easier and they can easily be provided with mechanical support. With larger aperture the resolving power of telescope increases.

46)

If the final image is formed at the distance of distinct vision, the magnifying power of the telescope is given as

$$m = \frac{f_o}{-f_e} \left( 1 + \frac{f_e}{D} \right)$$

Astronomical telescope in near point adjustment.



For an astronomical telescope, we will select lens of power 10D for eye piece and lens of power 0.5 D for objective

because magnifying power of telescope is  $m = \frac{f_o}{f_e}$

Resolving power of the telescope:

Given:  $D_o = 6 \text{ cm}$ ,  $\lambda = 540 \times 10^{-9} \text{ m}$

$$\text{R.P.} = \frac{D_o}{1.22\lambda} = \frac{6 \times 10^{-2}}{1.22 \times 540 \times 10^{-9}}$$

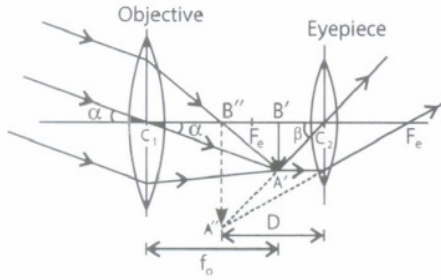
$$= 0.9 \times 10^5$$

47)

If the final image is formed at the distance of distinct visioa, the magnifying power of the telescope is given as

$$m = \frac{f_o}{-f_e} \left( 1 + \frac{f_e}{D} \right)$$

Astronomical telescope in near point adjustment.



Magnifying power or angular magnification of an astronomical telescope is defined as the ratio of the angle subtended at the eye by the final image at the least distance of distinct vision to the angle subtended at the eye by the object at infinity, when seen directly.

Let angle subtended by the object on the eye at C<sub>1</sub> is  $\alpha$ .

$$\angle A'C_1B' = \alpha$$

Further, let  $\angle A''C_2B'' = \beta$ , where

$$C_2B'' = D$$

By the definition, magnifying power,

$$m = \frac{\beta}{\alpha} \dots\dots(i)$$

As the angles  $\alpha$  and  $\beta$  are small, therefore,  $\beta \approx \tan\beta$  and  $\alpha \approx \tan\alpha$

$$\text{From (i), } m = \frac{\tan \beta}{\tan \alpha} \dots\dots(ii)$$

$$\text{In } \Delta A'B'C'_2, \tan\beta = \frac{A'B'}{C_2B'}$$

$$\text{In } \Delta A'B'C_1 \tan\alpha = \frac{A'B'}{C_1B'}$$

Putting in (ii), we get

$$m = \frac{A'B'}{C_2B'} \times \frac{C_1B'}{A'B'}$$

$$m = \frac{C_1B'}{C_2B'} = \frac{f_o}{-u_e} \dots\dots(iii)$$

where  $C_1B' = f_o =$  focal length of the objective lens and  $C_2B' = -u_e'$  distance of  $A'B'$ , acting as the object for eyepiece.

$$\text{Now, for an eyeplece, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Taking  $v_e = -D, u = -u_e$  and  $f = +f_e$ , we get

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e} \left( 1 + \frac{f_e}{D} \right)$$

Putting in (iii), we get

$$m = - \frac{f_o}{f_e} \left( 1 + \frac{f_e}{D} \right)$$